FAN, TA'LIM VA AMALIYOT INTEGRATSIYASI

ISSN: 2181-1776

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SOLVING PHYSICS PROBLEMS USING DIMENSIONAL ANALYSIS

Key words: Physics problem solving, dimensional analysis, new approach, methods of problem solving.

Abstract

There are many different types of ways of solving physics problems. All of them have certain features. The successful solution of a particular problem in physics largely depends on the correct choice of the applied method for this task. In this paper, one of such methods is proposed, a method of analyzing the dimensions of physical quantities, which can help students solve some physical problems. The essence of the method is that when solving the problem, if it is previously known what physical quantities are involved in the process under study, it is possible to establish the nature of the dimension that relates these quantities by comparing the dimensions. The method is very fruitful in those cases where finding the desired pattern directly meets significant mathematical difficulties or requires knowledge of such process details that are not known in advance. For a deeper understanding of the method, we presented four examples of solving problems that can be found in physics textbooks.

Introduction

When a student tries to solve a physics problem, there is almost always a desire to simplify it extremely. Getting to the calculations, the student not only tries to imagine what he wants to calculate, but also tries to understand what will result from the work. The set of techniques for creating a preliminary picture of the phenomenon under study, which is used in physics, is usually called qualitative methods (Mualem & Eylon, 2007). These include analysis of dimensions, estimates using simple models and finally, the use of symmetry properties, conservation laws, and the prohibitions that follow from them (Shute, 2013). In this paper, we consider a method for analyzing dimensions and solving some physics problems using this method. Our aim is to describe the method for instructors and recommend them the use of this method as a way of problem solving especially for talented students.

The expression of derived units of measurements in terms of the basic unit of measurement is called its *dimension* (Tirskii, 2001). Dimensional analysis is a method of establishing a relationship between physical quantities that are essential for the phenomenon under study, based on a consideration of the dimensions of the units of the quantities (Pescetti, 2008). It is based on the requirement: the equation expressing the desired relationship must remain valid for any change in the units of the quantities included in it. If this requirement is met, then the dimensions on the left and right sides of the equation coincide. For example force must be equal to Newton, which can be represented with combinations of time, length and mass i.e. $F = M*L*T^{-2}$ (M – mass, L – length, T - time). If this does not happen, then a change in a physical quantity will cause different changes in both sides of the equation and the equality will be violated. Inequality in the dimensions of the left and right sides of the equation can mean that some quantity that is significant for a given phenomenon is not taken into account, or the equation must include an unaccounted dimensional constant (Bhaskar & Nigam, 1990).

Solving a theoretical problem using dimensional analysis makes it possible to find practical application of the solved problems and evaluate its real life application (Bohren, 2004). Dimension analysis can also be used as an everyday tool for checking equations. This method helps to obtain particular solutions to problems that are too complicated to solve by conventional methods (Tsaparlis, 1998).

The advantages of the method can be seen by considering various examples solved using this method (Lira, 2013). But despite this, the dimensional analysis method does not replace the theoretical method, which allows student to accurately solve a problem, since the theoretical method makes it possible to penetrate deeper into the essence of the problem (Craig, 2003). Although, with the help of dimensional analysis, you can evaluate the task, it is easy to check the equations obtained for correctness and enrich mathematical skills.

Methodology

In this paper, we will consider the practical application of the dimensional analysis method. Since, the general method for constructing dimensionless complexes and determining the dependencies between different quantities is easiest to immediately illustrate with specific examples. Also, the point is not only that such examples, by virtue of their clarity, make it possible to more clearly understand the essence of the method. It is also very important that intuition plays a large role in the methods of applying dimensional theory, and therefore it is not always possible to act within the framework of formal rules. Of course, the formal application of the method of the theory of dimensions always gives the correct results, but not always exactly what interests us.

We will discuss particular examples that will give a clear idea of the method. It is hoped that for students who deal with this method for the first time it will give some skills in applying the dimensional method as a tool in further problem solving or in scientific research.

To solve certain problems, it can be brought the following statements of evidence which can be found in the literature (Georgi, 1993; Curtis et al., 1982): 1. As the main physical quantities we take the length, mass and time with the corresponding notations.2. The dimension of an arbitrary physical quantity can only be the product of the degrees of the dimensions of the quantities taken as the main ones. 3. The dimension of both parts of the equality, reflecting some regularity, should be the same. 4. Any relationship between a certain number of dimensional quantities characterizing a given physical phenomenon can be represented as a ratio between a smaller number of dimensionless combinations composed of these quantities. The last statement is called the Pi-theorem (Hanche-Olsen, 2004). The application of the Pi-theorem (Buckingham theorem) for these purposes reveals the main dimensionless parameters characterizing the phenomenon under consideration. When constructing a dimensionless combination, some quantities will have to be raised, of course, to some extent. Equating the composed dimensionless combination to a constant, we obtain the desired regularity.

We included four problems and their solutions from diverse physics topics in this paper.

Examples

Problem 1.

As the first example, we took one of the most common problems that can be found in physics handbooks. Figure 1. shows a bar system with mass *m* and a spring. There is no friction force between the stand and the bar. The spring with the bar was extended by *h* distance from the state of equilibrium (point O) and released. When the spring is extended by *h*, an elastic force F arises, which tends to return the spring to its original position. Determine the time t of the return of the bar to its equilibrium position.

Figure 1.

Solution 1.

We solve the problem in the following way: we need to provide time t as some function of the quantities m, h, and F known for this problem. Suppose that this function may include exponential functions, trigonometric functions, or other non-algebraic functions for which the arguments can only be dimensionless quantities. From the condition of the problem, it can be seen that in the system of units L - length, M - mass, T - time, it is impossible to make any dimensionless combination of m, h and F, whose dimensions are respectively M, L and LMT^{-2}, since T is included only in the dimension of force. Therefore, force cannot enter such a combination, and m and h cannot give a dimensionless combination. The only possible form of connection between t with m, h and F is an algebraic function. We can search for this function in the following form

$$t = CF^a h^b m^d \tag{1.1}$$

where C is an unknown dimensionless coefficient of proportionality, and a, b and d are unknown exponents. Equating the dimensions of the left and the right sides of expression (1.1), we obtain the following expression:

$$T = L^a M^a T^{-2a} L^b M^d \tag{1.2}$$

If the exponents of equation (1.2) with the corresponding symbols of the dimension of the basic units in the left and right sides are equal, then this equation will be invariant with respect to the basic units.

$$0 = a + b, \qquad 0 = a + d, \qquad 1 = -2a. \tag{1.3}$$

From these expressions it can be obtained that

$$a = -\frac{1}{2}, \qquad b = \frac{1}{2}, \qquad d = \frac{1}{2}, \tag{1.4}$$

correspondingly

$$t = C\sqrt{\frac{mh}{F}}\ . \tag{1.5}$$

The analysis does not allow anything about the value of the coefficient C, this is the main drawback of the method used. If the force F is proportional to h, then we can write expression (1.5) as follows:

$$t = C\sqrt{\frac{m}{k}}\ . \tag{1.6}$$

Where k is the spring constant. From this we can conclude that time does not depend on elongation - *h*. If we solve this problem by the traditional method, then we can get the same solution as in (1.6) but with the coefficient C = π / 2.

Problem 2.

Let us try to solve one more easy problem analogously to the previous problem which is related to the second law of Newton. An object of mass *m* is affected by a constant force *F* on the path L. Determine the speed that the object acquires at the end of the path.

Solution 2.

As in the previous task, we write the equation for speed in the form:

$$\vartheta = CF^a h^b m^d \tag{2.1}$$

Equating the dimensions of the left and right sides of formula (2.1), we obtain the following expression:

$$LT^{-1} = L^a M^a T^{-2a} L^b M^d \tag{2.2}$$

Comparing exponents, we easily obtain

$$a = \frac{1}{2}, \qquad b = -\frac{1}{2}, \qquad d = \frac{1}{2}. \tag{2.3}$$

Correspondingly

$$\vartheta = C\sqrt{\frac{Fh}{m}} \ . \tag{2.4}$$

The solution of the problem by the traditional method gives

$$\vartheta = \sqrt{\frac{2Fh}{m}} \ . \tag{2.5}$$

From this it follows that the meaning of the constant is $C = \sqrt{2}$

Problem 3.

The circular platform rotates around a vertical axis with an angular velocity ω (see. Fig. 2). On the platform there is a ball of mass m attached to the axis by a thread whose length is L. Determine the tension of the thread F at the moment of separation of the ball from the platform.

Figure 2.

Solution 3.

To solve this problem, as in the previous tasks, we first write the desired value through the known values:

$$F = Cm^a\omega^bL^d \tag{3.1}$$

Equating the dimensions of the left and right sides of formula (3.1) we obtain the following expression:

$$MLT^{-1} = M^aT^{-b}L^d \tag{3.2}$$

From (3.2) we can find the values of exponents:

$$a = 1, \qquad b = 2, \qquad d = 1. \tag{3.3}$$

Substituting the obtained values into expression (3.1), we can obtain the following

$$F = Cm\omega^2L \tag{3.4}$$

Solving this problem by the traditional method, we can get the answer $F = m\omega^2L$. In this case, the constant C = 1. Thus, we obtained the exact answer to the problem using the dimensional analysis method.

Problem 4.

Along with these tasks, the dimensional analysis method can help us to easily find solutions even some complex physics problems (Kazakova, 2010). Next problem can be example of this.

In an atomic explosion, an instantaneous release of significant energy E occurs in a small region. A strong spherical shock wave arises in the explosion region. The pressure behind the front of the shock wave at the initial stage of the explosion is many thousand times greater than the initial air pressure, whose influence on the wave propagation process can be neglected (Sedova, 1972). Let us try to find the law of propagation of a shock wave R = R (t).

Solution 4.

The parameters in this problem are the energy *E*, the initial density of unperturbed air *ρ*, and time *t*. To solve the problem, we write the desired function in the following form:

$$R(t) = CE^at^b\rho^d \tag{4.1}$$

Equating the dimensions of the left and right sides of formula (4.1), we write:

$$L = M^aL^{2a}T^{-2a}T^bM^dL^{-3d} \tag{4.2}$$

Equating the exponents from the left side to the right side, we obtain:

$$a = 1/5, \qquad b = 2/5, \qquad d = -1/5. \tag{4.3}$$

Substituting the obtained values into expression (4.1), we can obtain the following

$$R(t) = CE^{1/5}t^{2/5}\rho^{-1/5} \tag{4.4}$$

The value of constant *C* can be found by solving the problem numerically corresponding to the gas-dynamic (mathematical) problem of a strong explosion (Sedov, 1972), or it can be found conducting an experiment with measuring the function R = R (t) at different times at a known charge energy E and density ρ. The value of this constant was calculated in the publications of Taylor (1950) it turned out to be approximately $7.14 \cdot 10^{20}$ erg.

Conclusion

The dimensional analysis algorithm proposed in this article can be used in the school physics program as an alternative way of solution of some physics problems. The dimensional analysis method gives us solution of problem with accuracy up to a dimensionless constant. The superiority of the method is that the student himself opens up new formulas, analyzes the physical process and evaluates the correctness of the answer received, since the mathematical apparatus of the method allows us to establish the incorrectness of the solution of physical task associated with the choice of a list of controlled parameters. Thus, the solution to the educational problem of physics is approaching the style of solving engineering and scientific problems. Another advantage of the method is the fact that students using this method can come to new levels of generalization and analogies that cannot be revealed by solving other problems of physics.

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Figures

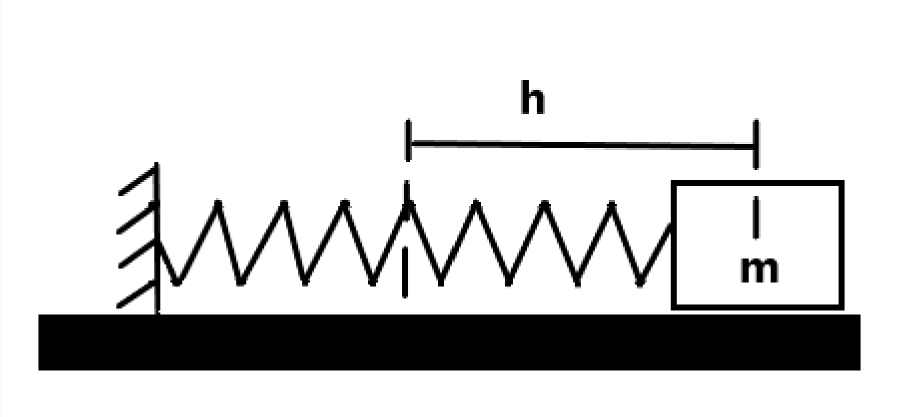

Figure 1.

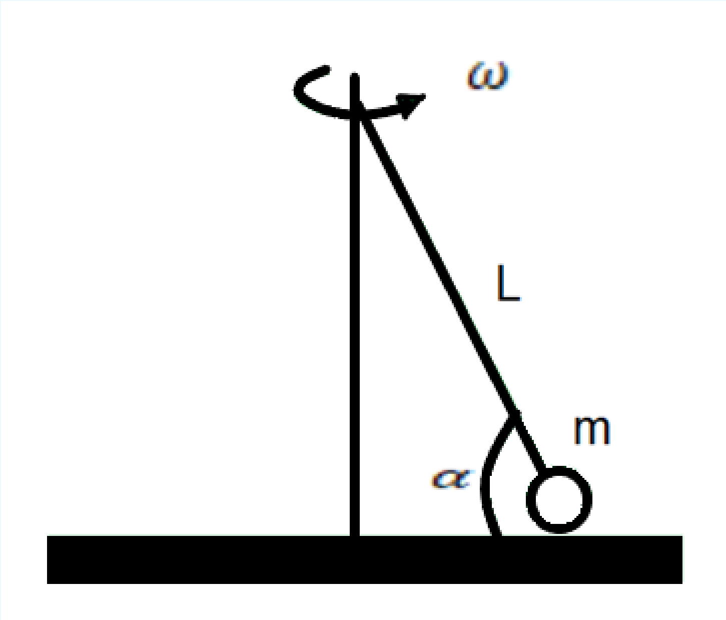

Figure 2.